MATH 464 (THEORY OF PROBABILITY)
HOMEWORK 9

SPRING 2019

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(1) The random variable Y is said to be obtained from the random variable X by *truncation* at the point a if

$$Y(\omega) = \begin{cases} X(\omega), & \text{if } X(\omega) \leq a \\ a & \text{if } X(\omega) > a \end{cases}$$

Write the distribution of Y in terms of the distribution of X.

(2) Given the following density function of a random variable X.

$$f_X(x) = \begin{cases} \frac{1}{\pi\sqrt{x(1-x)}}, & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the distribution function of X, i.e., $F_X(x)$.
(b) Show that $\mathbb{E}(X) = \frac{1}{2}$.

(3) The random variable X has density function

$$f(x) = cx(1-x), \qquad \text{for } 0 \leq x \leq 1.$$

Determine the value of c, and find the mean and variance of X.

(4) If $Z \sim \mathcal{N}(0,1)$. Find the mean and variance of $Y = e^{2Z}$.

(5) Let $X_1, X_2, \ldots, X_n$ be independent identically distributed (i.i.d.) random variables from $U(0,1)$. Denote $V = \max\{X_1, \ldots, X_n\}$ and $W = \min\{X_1, \ldots, X_n\}$.
(a) Find the distributions and the densities and the distributions of each of V and W.
(b) Find $\mathbb{E}(V)$ and $\mathbb{E}(W)$.

(6) Let $Z \sim \mathcal{N}(0,1)$.
(a) Find the density of $Y = |Z|$.
(b) Find $\mathbb{E}(Y)$.

(7) Suppose $X \sim \mathcal{N}(0,1)$. Use integration by parts to show that $\mathbb{E}(X^k) = (k-1)\mathbb{E}(X^{k-2})$. Derive that $\mathbb{E}(X^k) = 0$ for all odd $k \geq 1$. Compute $\mathbb{E}(X^4)$ and $\mathbb{E}(X^6)$. Derive a general formula for $\mathbb{E}(X^{2k})$.

(8) Let $X \sim \exp(1)$, find the density function of $Y = (X-2)/(X+1)$.

(9) Find the mean and variance of the Gamma(λ, ω) distribution.